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IN THREE VOLUMES.

VOL. II.

NON SIBI, SED PATRIÆ. CATO.

By WILLIAM HENRY HALL, Esq. *Millman Place, Bedford Row, London.*

AND OTHER GENTLEMEN OF SCIENTIFIC KNOWLEDGE, WHOSE NAMES AND ADDRESSES APPEAR IN THE WORK.

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A TREATISE ON DIALLING.

PART I. SECT. I.

DIALLING is the Art of drawing Dials on any given plane, or on the surface of any given body.

The Greeks and Latins named this art *gnomonica*, and *sciatherica*, because through it they were taught to discover the hour of the day by the shadow of a gnomon. Some have named it *photosciatherica*, for this reason, that the hours are commonly exhibited by the *light*, or rather the *shade*, of the sun. Others have called the art *horologiography*; and others again *sciographia*, because it treats of shadows.

It is uncertain, who was the first that found out the art of constructing dials. Some attribute the invention to Anaximenes Milesius, who lived about 556 years before Christ; others, to Thales, who flourished about the same time, or rather earlier. But to leave the opinions of different authors, I think it is evident, that the art of making dials was known long before the days of either of these philosophers. Witness, the dial of Ahaz, mentioned in Isaiah, chap. xxxviii. ver. 8. which must have been at least 150 years prior to the date abovementioned. It appears, however, that the Romans were unacquainted with dials till Papirus Cursor, about 260 years before the Christian æra set up a dial at Rome, prior to which, says Pliny, there is no mention of any account of time, other than by the sun's rising and setting.

Clavius is said to be the first professed writer on this subject. The art has been since handled by Dechales, Ozanam, Wolfius, M. Picard, De la Hire, Sturmius, Paterson, Michael, Leybourn, Leadbetter, Muller, Ferguson, and many others. But among all the books on dialling, none are more deservedly esteemed than that of J. Wells, Esq. printed at London in 1635, under the title of "The Art of Shadows," and that excellent Treatise of the late Mr. Emerson, printed in 1770.

There are various methods of drawing dials. Some find all the requisites by means of a terrestrial globe, and then transfer them to the dial plane, but in the construction of large dials, where accuracy is required, this method is too vague and uncertain. Others find the requisites by trigonometrical calculations, and insert them in a table for use, and so taking off the due measure of each from a plain scale of proper size, they easily and accurately form the several lines on the dial plate.

Others, again, construct from a *dial scale* fitted to the particular latitude of the place; but, as in the practice of making dials a variety of latitudes will occur, and it would be exceeding difficult, nay, almost an endless task, to procure such scales for all latitudes; or if procured, as the advantage would never compensate for the expence, this method is not fit for general use, and of course is not worth notice in this place, as the œconomy of our plan is cautiously to avoid extraneous matter and things of small importance, while we wish to be copious, and to afford our readers every satisfaction in those articles which appear to be of real utility.

Another method of constructing dials is by lines drawn purely geometrical, without the help of calculation, and this method being more mechanical and scientific than the rest, would be the best for practice; was it not that in some cases the lines become so numerous, and their intersections so many, as to render it both tedious and inaccurate.

In what follows, I shall therefore prefer that method by the scale with the help of calculation, it being the most general, correct, and expeditious; not, however, omitting the geometrical method, in cases where it is well adapted for actual use.

Before I enter into the practice of describing dials, it will be necessary to treat of the gnomonic projection of the sphere.

SECT. II.

GNOMONIC PROJECTION OF THE SPHERE.

IN the gnomonic projection, the plane projected on, is supposed to touch the hemisphere to be projected, in its vertex; the point of contact being the centre of projection. But a projection may be made nothing differing from this, upon any plane parallel to such touching plane; for it is only taking a greater or less radius of projection, according to the greater or less distance, which is in effect projecting a greater or less sphere upon its touching plane.

When the sphere is to be projected this way upon a given plane, it will assist the imagination to suppose the eye placed in the centre of the sphere and looking on the plane whose position is given; as by that means, the circles of the sphere, as projected on the plane, will appear in due order.

In this projection, every great circle as BAD (*Fig.* 1.) is projected into a right line, perpendicular to the line of measures, and at a distance from the centre equal to the cotangent of its inclination, or the tangent of its nearest distance from the pole of projection.

For let CBED be perpendicular both to the given circle BAD and the plane of projection, then will the intersection CF be the line of measures. Now since the plane of the circle BD and the plane of projection are both perpendicular to BCDE, the common section will also be perpendicular to BCDE, and consequently it will also be perpendicular to the line of measures CF. Now since the projecting point A is in the plane of the circle; all the points of the circle will be projected into that section; that is, into a right line passing through d, and perpendicular to Cd. And Cd is the tangent of CD, or cotangent of CdA.

Hence a great circle perpendicular to the plane of projection, is projected into a right line; passing through the centre of projection, and any arch is projected into its correspondent tangent. Thus the arch CD is projected into the tangent Cd.

Also any point as D, or the pole of any circle, is projected into a point d distant from the pole of projection C, the tangent of D's distance from the said pole.

If two great circles be perpendicular to each other, and one of them passes through the pole of projection, they will be projected into two right lines perpendicular to each other.

Therefore, if a great circle be perpendicular to several other great circles, and its representation pass through the centre of projection, then all these circles will be represented by lines parallel to one another, and perpendicular to the line of measure or that first circle.

If two great circles intersect in the pole of projection, their representations shall make an angle at the centre of the plane of projection, equal to the angle made by those circles on the sphere.

Any less circle parallel to the plane of projection, is projected into a circle, whose centre is the pole of projection; and radius, the tangent of the circle's distance from the pole of projection. Hence, if a circle be parallel to the plane of projection, and 45° from the pole, it is projected into a circle equal to a great circle of the sphere, and therefore it may be looked upon as the primitive circle in this projection, and its radius esteemed, the radius of projection.

Every less circle (not parallel to the plane of projection) is projected into a conic section, whose transverse axis, is the line of measures, and whose nearest vertex is at a distance from the centre of the plane, equal to the tangent of its nearest distance from the pole of projection; the other centre is distant therefrom equal to the tangent of its farthest distance from the pole of projection.

For let BE (*Fig.* 2.) be parallel to the line of measures dp, so will any circle be the base of a cone whose vertex is A, and therefore such cone being produced will be cut by the plane of projection in some conic section. Thus the circle whose diameter is DF, will be cut by the plane in an ellipsis, whose transverse axis is df; and Cd is the tangent of CAD, and Cf of CF. In like manner, the cone AFE being cut by the plane f, will be the nearest vertex, and the other point in which E is projected is at an infinite distance. Also the cone AFG (whose base is the circle FG) being cut by the plane f, is the nearest vertex, and GA, being produced, gives d the other vertex.

It follows from what has been said, that if the distance of the farthest point of the circle be less than 90° from the pole of projection, it will be projected into an ellipsis.

If the farthest point be more than 90° from the pole of projection, it will be projected into an hyperbola. Thus the circle FG is projected into an hyperbola, whose vertices are f and d, and transverse fd.

In the circle EF, where the farthest point E is just 90° from C, it will be projected into a parabola, whose vertex is f.

Also if H be the centre, and K, k, l, the focus of the ellipsis, hyperbola, or parabola, then $HK = \frac{Ad - Af}{2}$, for the ellipsis; $Hk = \frac{Ad + Af}{2}$ for the hyperbola; and (drawing fn perpendicular to AE) $fl = \frac{nE + Ff}{2}$, for the parabola, which are the representations of the circles DF, FG, and FE respectively.

These things being premised, I proceed to the practical part of projection, in which, for the sake of brevity and utility, I shall confine myself as much as possible to the method by the plain scale, the other methods which depend upon rigid geometrical constructions, being in some cases exceeding tedious, and on account of the variety of lines and intersections that occur, the figures when constructed are liable to far greater inaccuracy than when they are drawn at once from a correct scale of chords, sines, tangents, &c.

ART. I. *To draw a great Circle, through a given Point, and at a given Distance from the Pole of Projection.* (*Fig.* 3.)

RULE. With the radius of projection describe the circle ABD, and with the tangent of the circle's distance from the pole of projection C, describe the circle EIF, and draw PK to touch this circle, so will PIK be the circle required.

ART. II. *To draw a great Circle perpendicular to a given Circle, that passes through the Pole of Projection, and at a given Distance from that Pole.* (*Fig.* 3.)

RULE. The primitive circle ABD being drawn in the given circle CI, set the tangent of the given distance from C to I; through I draw Kp perpendicular to CI, and it will be the representation required.

ART. III. *To measure any Part of a great Circle or to set any Number of Degrees thereon.* (*Fig.* 4.)

RULE. Let EP be the great circle, EBD be a circle described with the radius of projection, and let IP be a part of the great circle; draw HD perpendicular to EP; apply CI to the tangents, and set the semitangent of its complement from C to A, which will be the dividing centre of EP; and drawing AP, the

the angle IAP will consequently be the measure of the given arch IP.

If the degrees be given, make the angle IAP equal thereto, which, (by drawing AP to meet the circle EP,) will cut off IP, the arch correspondent thereto.

ART. IV. *To draw a great Circle so as to make a given Angle with a given great Circle, at any assigned Point; or to measure an Angle made by two great Circles.* (*Fig.* 5.)

RULE. Let P be the given point, and PB the given great circle. Draw through P, and C, the centre of projection, the line PCG; to which from C, draw CA perpendicular, and equal to the radius of projection. Draw PA and AG perpendicular to it; at G, erect BD perpendicular to GC, cutting PB in B; draw AO bisecting the angle CAP. Then at the point O make BOD equal the given angle, and from D draw the line DP; so will BPD be the angle required. Or, if the degrees in the angle BPD be required; from the points BD, draw the lines BO, and DO, and the angle BOD will be the measure of BPD.

COR. If an angle be required to be made at the pole or centre of projection, equal to a given angle, this will be effected by drawing two lines from the centre, making the angle required.

ALSO, If one great circle is to be drawn perpendicular to another great circle, it must be drawn through its pole.

ART. V. *To project a less Circle parallel to the Primitive.*

RULE. With the radius of projection AC (*Fig.* 3.) and centre C, describe the primitive circle ADB, and with the radius CE, equal to the tangent of the circle's distance from its pole, describe the circle EFG for the circle required.

ART. VI. *To draw a less Circle perpendicular to the Plane of Projection.* (*Fig.* 6.)

RULE. Through the centre of projection C, draw its parallel great circle TI; take the tangent of the circle's distance from its parallel great circle, and set it from C (the centre of projection) to V, and the secant thereof from C to F. Then with the semitransverse CV, and focus F, describe the hyperbola WVHK, which will represent the given circle.

ART. VII. *To project any less oblique Circle given.* (*Fig.* 7.)

RULE. Having drawn the line of measures *dp*, take the tangents of the circles nearest and farthest distance from the pole of projection, and set from C to *f* and *d*, which will give the vertices, and bisect *df* in H. Take half the difference or half the sum of the secants of the greatest and least distances from the pole of projection, and set from H to K or *k*, for the focus of the ellipsis or hyperbola, which may then be described; and it will represent the given circle.

ART. VIII. *To find the Pole of any Circle in the Projection, DMF.* (*Fig.* 8.)

RULE. From the centre of projection C draw the radius of projection C A perpendicular to the line of measures D F; apply D C and C F to the tangents, and set the tangent of *half the difference* of their degrees from C to P; if D and F lie on contrary sides of C; but, *half the sum*, if on the same side, and it will give P, the pole; or by ART. III. set off from D to P, the circle's distance from the pole, which will give P as before.

COR. If it be a great circle as B G (*Fig.* 9.) draw the line of measures G C and C A perpendicular to it, and equal to the radius of projection; make G A P a right angle, so will P be the pole required.

ART. IX. *To measure any Arch of a less Circle; or to set any Number of Degrees thereon.* (*Fig.* 9.)

RULE, Let C be the centre of projection, F*n* the arch, and P the pole of the given circle: apply C P to the tangents, and set the tangent of its half from C to O, and the cotangent of its half from C to D; with the radius G C taken equal to the tangent of the degrees in F P, (the given circle's distance from its pole) describe the circle GSH: then D*l* being drawn through *n* or *l* will cut off H*l* = F *n*. Or, O being found as before, erect O S perpendicular to C O; then through the given point *n* draw P Q *n*, and the angle Q C H will be equal F*n*: Or, apply C P to the tangents, and set the cotangent thereof from C to G. Erect G B perpendicular to G C. Through *n* draw P *n* B; and join B O, so will the angle G O B = F *n*.

COR. If the less circle be perpendicular to the plane of projection as V K K (*Fig.* 6.) it is only drawing the perpendiculars V C, H G, to its parallel great circle C I; and C G (measured by ART. III.) will be equal to V H; or the degrees set from C to G, will cut off V H equal thereto.

AN EXAMPLE illustrating the foregoing RULES.

Let it be required to project the Sphere on a Plane parallel to the Horizon, for July 31, 1787, at 10 in the Morning. Lat. 35½ N. (*Fig.* 10.)

With the radius of projection and centre C describe the primitive circle A D B. Through C draw the meridian P E, and A S perpendicular to it for the prime vertical. Set off C P = 35½, the latitude, so will P be the north pole. Draw P*p* perpendicular to C P, for the 6 o'clock meridian. Set the co-latitude from C to E, and draw E Q perpendicular to C E for the equinoctial. Make E B = 30° (or two hours the time from noon) and draw the 10 o'clock meridian P B. Set the sun's declination 18° 27′ from B to ☉, then will ☉ be the sun's place at 10 o'clock. Through ☉ draw the azimuth circle C Q. Also through ☉ may be drawn (by ART. VII.) a parallel to the equinoctial E Q for the sun's parallel of declination.

C O measured by ART. III. = 31° ¼, the complement of the ☉'s altitude; and the angle E C ☉ measured by ART. IV. = 65°, 10′, his azimuth.

In like manner may be drawn any other circles of the sphere, and the places of moon, stars, and planets exhibited.

PART II. SECT. III.

THE PRACTICE OF DRAWING VARIOUS SORTS OF DIALS.

This I shall endeavour to methodize, by reducing the different kinds to three classes, for all dial planes are either parallel, perpendicular, or inclining to the horizon.

DEFINITIONS.

1st. *Dial Planes*, are those on which dials are or may be described.

2d. *The Denomination of any Dial*, is taken from that great circle of the sphere, to which the dial plane is parallel.

3d. *Declination of a Plane*, is an arch of the horizon, contained between the plane and the prime vertical, or between the meridian and a plane perpendicular to the dial plane, and is always reckoned from the South or North.

4th. *Reclination of a Plane*, is the angle which it makes with a vertical plane.

5th. *Inclination of a Plane*, is the angle which it makes with the horizon.

6th. *The Centre of a Dial*, is the point where all the hour lines meet, or to which they tend.

7th. *The Style or Gnomon of a Dial*, is a pin, or piece of metal or wood, &c. raised perpendicular to the plane of the dial, the shadow of which, as an index, points out the hour of the day.

8th. *The Subtile*, is the line on which the stile is erected. This always passes through the centre of the dial.

9th. *The Stiles Height*, is the angle which the upper edge thereof makes with the substile; but this is to be understood when the stile is in the form of a triangle, and then the angular point is at the centre of the dial. If the stile is a pin, the length thereof is the height. If the stile is a parallelogram, the centre is at an infinite distance.

10th. *The Substiles Distance from the Meridian*, is the angle which it makes with the 12 o'clock line.

11th. *Meridian of the Plane*, is the meridian perpendicular to the plane of the dial; it is the same as the substile; but the meridian of the place is perpendicular to the horizon.

12th. *The Height of the Meridian*, is an arch of the great circle, or dial plane, comprehended between the horizon and meridian of the place.

13th. *Plane's Difference of the Longitude*, is the angle on the sphere, contained between the meridian of the place, and the meridian of the plane. This is also called the *Inclination of Meridians*.

14th. *Hour Arch*, is an arch of the Equinoctial answering to the time, or the angle at the Pole, thus 15° = 1 hour, 30° = 2 hours, 45° = 3 hours, &c. This takes its beginning at the substile.

15th. *Hour Angle*, is the angle which any hour line makes with the substile.

16th. *Horizontal Line*, in any dial, is a line drawn parallel to the plane of the horizon, and is made by the horizontal plane's cutting the dial-plane, and passing through that point of the stile, whose shadow shews the hours.

17th. *The Equinoctial Line*, is made by the intersection of the plane of the Equinoctial and Dial Plane.

18th. *An Azimuth Line*, is the Intersection of an Azimuth with the dial plane; and so for other lines of the like kind.

19th. *The Contingent Line*, is a line drawn through the foot of the stile perpendicular to the substile. It serves instead of the equinoctial for finding the hour points, through which the hour lines are to be drawn. When the contingent does not pass through the foot of the stile it represents the equinoctial.

SECT. IV.

PROPOSITIONS.

Mr. Emerson has demonstrated the following Propositions respecting dials.

PROPOSITION I. If a right line be fixed any where upon the earth, parallel to the earth's axis, the shadow thereof by the sun, will move uniformly about it, describing equal angles in equal times.

2d. If a line be elevated above a plane and fixed there, its shadow will be the same at the same moment of time, in whatever part of the earth it is placed, provided it be in a parallel situation.

3d. Hence a dial removed from its true situation to any other and placed in a situation quite parallel, will, when the sun shines, always shew what a clock it is at the place it came from.

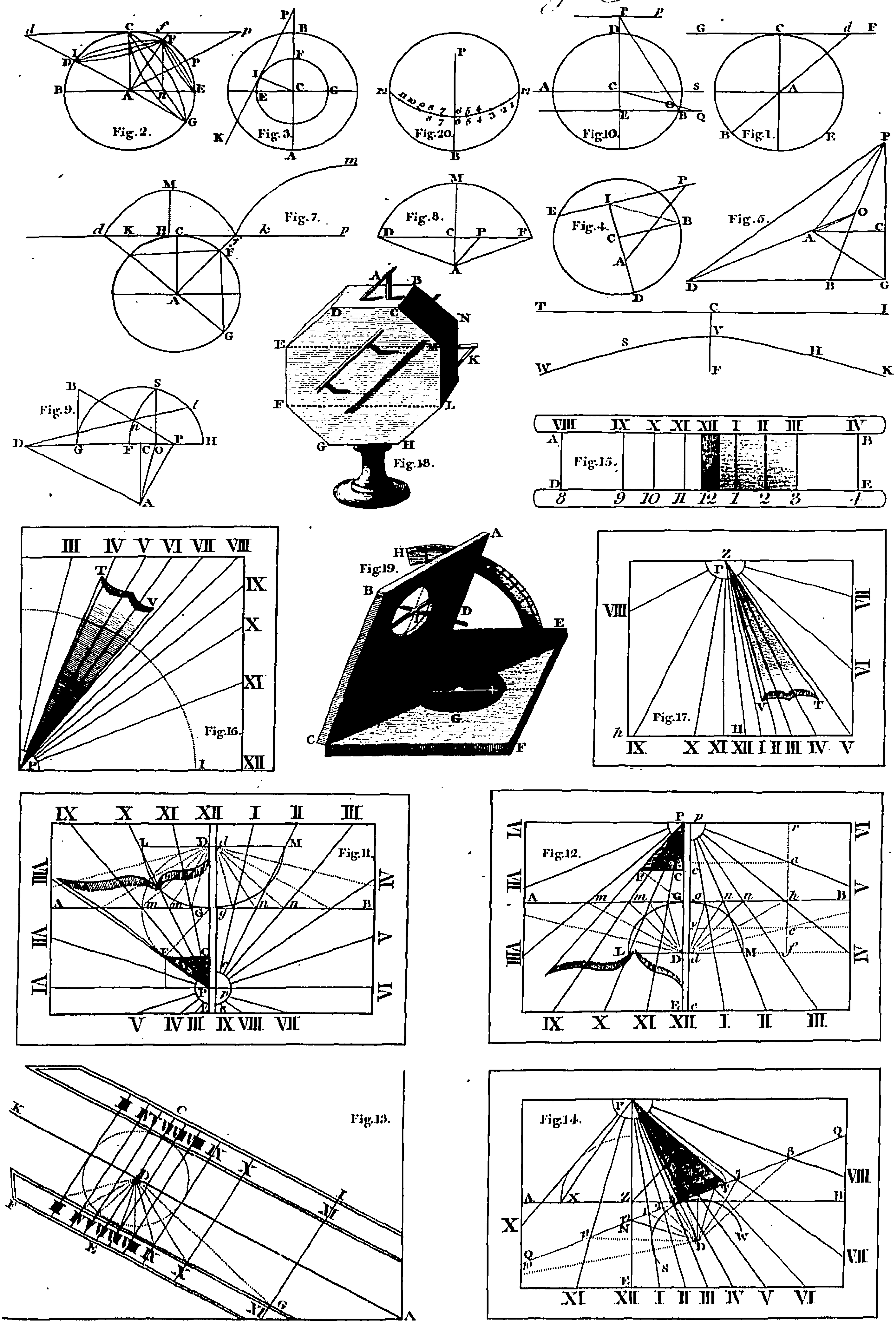
4th. In every dial that has a stile, the edge of it that gives the shadow must always be parallel to the earth's axis, and point directly to the two poles.

5th. In any dial, the angle of the stile's height above the substile is equal to the height of the pole above the plane of the dial.

6th. The

Various Dials with the Gnomonic Projection of the Sphere.

See Treatise on Dialling.



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6th. The interfection of the meridian of the place and the plane of the dial is always the hour line of 12 o'clock; and all the other hour lines are the interfections of the feveral meridians or hour circles, with the plane of the dial, all paffing through the ftile.

7th. In a dial with a centre, if the hour lines be produced through the centre, fo as to appear on the other fide of the plane; and if the ftile be alfo produced through the plane, there will be a dial formed on the other fide of the plane, in which the fame lines produced refer to the fame hours.

8th. In all upright dials the hour line of 12 is perpendicular to the horizon.

9th. In a direct eaft or weft reclining dial the hour line of 12 is parallel to the horizon.

10th. If any erect declining dial be removed along the plane of its great circle, till its difference of longitude be equal to the planes difference of longitude or inclination of meridians, it will become a full fouth or north erect dial.

11th. A reclining eaft or weft dial plane, will be an upright plane under the fame meridian at 90° diftance, and in a parallel fituation.

SECT. V.

DIALS PARALLEL TO THE HORIZON.

Under this defcription are thofe dials that are termed horizontal, their planes being always fixed parallel to the horizon, through which, if obftacles do not intervene, the fun fhines upon them on each day of the year from his rifing to his fetting, whatever may be the latitude of the place, which is not the cafe with any other kind of dials; and therefore this fort of all others is of the moft general ufe.

Art. X. *To conftruct an horizontal Dial for any given Latitude.*

1ft. Geometrically. Take any Point as P (*Fig.* 11.) for the centre, through P draw the meridian DE; at P make the angle DPR equal to the latitude. Draw a line parallel to DE, at a diftance equal to the perpendicular height of the ftile, cutting PR in F, fo will FC be the height, and C the foot of the ftile. Draw FG perpendicular to FP. Through G draw the contingent line AGB perpendicular to DE, which in this cafe reprefents the equinoctial. — Let G*g* be the thicknefs of the ftile, and draw *dge* parallel to DGE. From G take GF, the neareft diftance to PR, and fet from G to D, and from *g* to *d*; and from the centres G and *g* with any radius defcribe two quadrants, and divide each into fix equal parts, or hours. Through the points of divifion draw lines from the centres D and *d*, interfecting AB in the hour points *m*, *m*; *n*, *n*, &c. then from the centres P and *p* draw lines through the points *m*, *n*, &c. and they will be the hour lines, which may be circumfcribed either by a circle, fquare, or any other figure, at the pleafure of the artift. The 6 o'clock hour line is drawn parallel to AB, and the hours before and after 6 are had by producing the oppofite hour lines through the centre.—Alfo PFC will be the form of the ftile which may be longer at pleafure, by producing FP.—The ftile muft be placed perpendicular to the dial plane, one edge thereof being on PG, and the other on *pg*; fo will the upper edge fhew the fhadow.

If no fpace is allowed for the thicknefs of the ftile, the lines DP and *dp*, will coincide; D and *d* will be one point, as alfo P and *p*. The two quadrants will fall into one femicircle, and the hour lines will all iffue from one point. But if the dial is thus conftructed, the ftile muft be made either exceeding thin fo as to occupy no fenfible fpace, or the upper edge thereof muft be made fharp, and the ftile fo fixed, that fuch edge may be exactly perpendicular to the fubftile.

Inftead of the triangular gnomon PFC, there may be a perpendicular pin, as FC, and then the fhade of the top F, will fhew the hour; or the ftile may be a plate of any form, with a hole in it, at F, perpendicular to C, for the fun to fhine through.

If half hours, or quarters, or ftill fmaller meafures of time are to be fhewn by the dial, then every hour in each quadrant muft be divided into fo many equal parts as are the number of fuch meafures in an hour; and lines muft be drawn from the centres D and *d* interfecting AB, and then lines drawn through fuch interfections from P and *p* as before. To fave trouble, after the hour lines are drawn on one fide of the dial, they may be transferred to the other fide. The hours on the Eaft fide muft be numbered 1, 2, 3, &c. from PD or XII, for the afternoon hours: and thofe on the Weft fide muft be numbered 11, 10, 9, &c. for the *forenoon* hours.

2d. By calculation of the plain scale.—The Latitude of the place being given, the only requifites to be found are the hour angles, which are had by the following proportion. As radius : fine of latitude : : tangent of hour-arch ; tangent of hour angle.

The hour arches are had by allowing 15° for each hour; $7\frac{1}{2}$° for half an hour, 3° 45′ for a quarter of an hour, &c. See Definition XIV.

In the latitude of London = 51° 32′, the hour angle anfwering to the fpace of 1 hour = 11° 51′; for two hours = 24° 19′; for 3 hours = 38° 4′; for four hours = 53° 36′; for 5 hours = 71° 6′; and for 6 hours (in all latitudes) 90°.

Thefe being known, take any point P for the centre, and draw PD for the meridian. Take P*p* = to the thicknefs of the ftile, and draw *pd* parallel to PD. Make the angles DP XI, and *dp* each = 11° 51′, for the hour lines of XI. and I; the angles DPX, and *dp* III, each = 24° 9′, for the hour lines of X and II. The angles DP IX, and *dp* III = 38° 4′, for the hour lines of IX and III, and fo for the reft. — Alfo the angles for the half hours, and quarters of hours, muft be found by the foregoing proportion, and then laid down as above. — If fmaller divifions than quarters of hours are required, it will generally be near enough, after the quarters are drawn, to divide them into as many equal parts as you pleafe, without further trouble of calculation.

Laftly, make the angle DPR = to the latitude for the ftile or gnomon.

I have been the more copious in explaining this Dial, that what follows may be the clearer underftood.

SECT. VI.

DIALS PERPENDICULAR TO THE HORIZON.

Under this defcription are all dials, whofe planes are vertical. — Thefe may be diftinguifhed into two kinds, viz. *direct*, and *declining*. — A direct dial is one that is drawn on a plane, facing either the Eaft, Weft, North, or South; and fuch a dial is accordingly named, an Eaft Dial, a Weft Dial, a North Dial, or a South Dial.

The Eaft and Weft Dials are parallel to, or in the plane of the meridian, and the North and South Dials are parallel to or in the plane of the prime vertical.

A Declining Dial is one that faces none of the cardinal points, but declines towards the Eaft or Weft. — They come under thefe our denominations, viz. A South-eaft Decliner, a South-weft Decliner, a North-eaft Decliner, and a North-weft Decliner.

Art. XI. *To conftruct Dials perpendicular to the Horizon.*

Case I. *To draw a direct Dial, as for Inftance, an erect direct South Dial.*

1ft. Geometrically, Take the Point P (*Fig.* 12.) near the top of the plane for the centre of the dial; through P, draw the meridian PE; at P make the angle EPR equal to the complement of the latitude. Take any point G in the line PE, and draw GF perpendicular to PR; and FC perpendicular to GP, and C will be the foot of the ftile. Through G, draw the contingent line AB, perpendicular to PE, reprefenting the equinoctial. — Take G *g* equal to the thicknefs of the ftile, and draw *p g q* parallel to PGE. — Make GD and *g q* = GF, and from the centres D and *q* with any radius defcribe two quadrants LG and M*g*. Divide each quadrant into fix equal parts or hours. From the centres D *q* draw lines through the points of divifion, cutting AB in the hour points *m*, *m*; and *n*, *n*, &c. Then lines drawn through the centres P and *p*, through all the points *m*, *n*, &c. will be the hour lines. The half hours, &c. are drawn in the fame manner. A perpendicular pin FC, or, which is better, the triangle RPG will be the ftile, which muft be fet perpendicular upon the hour lines of XII, PG, and *pg*; the ftile PR pointing downwards towards the South pole.

When the lines do not interfect within the plane, as at the firft divifion towards L and M, bifect the line *q g* in *v*, and draw *vc* parallel to AB, cutting *q c* in *c*: through *c* draw *c h a* parallel to *p q*, and fet off *h a* = half *gp* drawing *p a* for the hour line.

N. B. The forenoon hours are on the left hand, the afternoon hours on the right.

2d. By calculation and the scale. — The proportion for finding the hour angle in this cafe is. As radius: cofine of latitude : : tangent of hour arch : tangent of hour angle. — In the latitude of London the hour angles for 11 and 1 is 9° 28′; for 10 and 2 = 19°, 45′; for 3, and 9 = 31° 53′; for 4 and 8 = 47° 8′; for 5 and 7 = 66° 42′, and for 6 (as ufual) = 90°. Alfo for the half hour, before and paft 12 = 4° 41′; for half paft 10, and half paft 1 = 14° 27′; for half paft 9 and half paft 3 = 25° 31′ &c. — Thefe being known:

Take any point P near the top of the plane for the centre of the dial; draw the meridian or 12 o'clock line PE, and *p e* parallel to it at a diftance equal to the thicknefs of the ftile. With the radius of 60° from a line of chords defcribe two quadrants from the centres P and *p*; fet off the feveral hour angles, both ways from the lines PE and *p e*, upon the quadrants, viz. 9° 28′ for 11 and 1; 19° 45′ for 10 and 2, &c. Through the points thus given draw lines from the centres P*p*, for the hour lines.

Laftly, Make the angle EPR = the complement of the latitude for the ftile to be fet upright upon the hour 12.

Case II. *To conftruct a Direct North Dial.*

The north plane lying in the fame azimuth with the fouth, but with its face the oppofite way, will have the fame hour angles as a fouth plane, only the meridian in this will reprefent the midnight hour inftead of noon; and the hours about it, 11, 10, 9, and 1, 2, 3, are quite ufelefs, becaufe the fun in his greateft north declination rifes but about a quarter before 4 and fets at a quarter paft 8. — Alfo the fun cannot fhine upon this plane longer than 21 minutes paft 7 in the morning, nor fooner than 21 minutes before 5 at night.

To make this dial, is but to turn the fouth dial upfide down, and leave out all the fuperfluous hours between 5 and 7, and 4

and

and 8; and the dial will be complete. There is no reaſon therefore why this ſhould be deemed a different kind of dial to the former, ſeeing that they both agree in the height of the ſtile and hour angles.

N. B. Thoſe hours to the weſt are the morning hours, and thoſe to the eaſt the evening hours.

Case III. *To draw a direct Eaſt Dial. (Fig. 13.)*

A direct eaſt dial is drawn upon the plane of the meridian facing the eaſt. The ſun only ſhines on this plane from his riſing till noon.

On the eaſtern ſide of the plane of the meridian draw a right line AB parallel to the horizon, and to this join AK, making with it an angle KAB, equal to the elevation of the equator: then, with the radius DE, deſcribe a circle, and through the centre D draw EC perpendicular to AK, by which means the circle will be divided into four quadrants. Each of theſe quadrants ſubdivide into ſix equal parts. And from the centre D through the ſeveral diviſions draw the right lines D IV, D V, D VI, D VII, D VIII, D IX, D X, D XI. Laſtly, in D erect a ſtyle equal to the radius D E, perpendicular to the plane; or, on two little pieces perpendicularly fixed in EC, and equal to the ſame radius DE, fit an iron rod, parallel to EC.

Thus will each index at the ſeveral hours project a ſhadow to the reſpective hour lines IV IV, V V, VI VI, &c.

By Calculation and the Scale.

Firſt find the diſtance of the hour lines from 6 o'clock, by this proportion. As radius is to the height of the ſtile in inches, ſo is the tangent of the hour arch from 6, to the diſtance of ſuch hour from the 6 o'clock line.

N. B. If you would have the dial plate juſt to include the 11 o'clock hour, then the height of the ſtile muſt be found by this proportion. As radius is to the length XI E, ſo is the tangent of 15° to the height of the ſtile.

Suppoſe the length of the ſtile to be three inches, then the hour diſtances from VI will be as follow, viz. The diſtance of V and VII. $\frac{4}{5}$ inch; of IV and VIII, 1.73 inches; of III and IX = 3 inches, the height of the ſtile; of X, 5.2 inches; of XI, 11.2 inches.

Theſe being known, and AB and AK being drawn as before, and DC for the 6 o'clock hour line, ſet off the ſeveral diſtances above ſpecified, and they will give the hour lines; alſo DC is the height of the ſtile, which may either be a perpendicular pin or a parallelogram of that height.

N. B. The ſtile muſt either be made exceeding thin on the top part, or a ſpace muſt be left for the thickneſs thereof as has been before ſhewn, and this obſervation is applicable to any other dial.

Case IV. *To draw a direct Weſt Dial.*

A direct weſt dial, is one deſcribed on the weſtern ſide of the meridian. The conſtruction of this dial is perfectly the ſame as that of an eaſt dial, only that its ſituation is inverted. If you look on the back ſide of the paper on which an eaſt dial is drawn, and hold it to the light, you will ſee the true figure of a weſt dial through the paper.

Art. XII. *To draw erect declining Dials.*

Case I. *To draw an erect South declining Dial.*

Geometrically. Let the dial decline to the weſt 36°, and let the latitude of the place be 54°½.

Take the point C, (*Figure* 14.) about the middle of the plane for the foot of the ſtile, through which draw the horizontal line AB, and make CF perpendicular to it, and equal to the perpendicular height of the ſtile. Make the angle CFZ equal to the declination (36°) to the right hand if it decline eaſt; or to the left if weſt, as in this caſe, cutting the horizontal line in Z. Through Z draw PZE perpendicular to AB for the meridian. Take ZF and ſet it from Z to X, in the line AB. Make ∠ZXP = latitude (54½) cutting the meridian in P, the centre of the dial. Through C and P draw PCD for the ſubſtile. Alſo through C draw QQ perpendicular to PD, cutting the meridian in N, for the contingent line. In CQ make CT equal to CF, and draw PT. Take the neareſt diſtance from C to PT, and ſet it from C to D, in the line PD, and draw DN. With the centre D and any radius as DC, deſcribe the ſemicircle SCW. Divide the ſemicircle into parts of 15° for hours. Through theſe points draw lines from the centre D interſecting the equinoctial QQ in the hour points XI, XII, I, II, III. From P, the centre of the dial, draw lines through the hour points, as P XI, P XII, P I, P II, P III, &c. for the hour lines. The ſtile may be either the perpendicular pin FC, or the triangle PCT ſet upright on the ſubſtile PC.

By Calculation and the Scale.

Here there are given the latitude = 54°½, and the declination of the plane = 36°, to find firſt the ſubſtile's diſtance, from XII; 2d, the height of the pole above the plane; and 3d, the plane's difference of longitude.

I. For the diſtance of the ſubſtile, the proportion is, as radius : ſin. declination (36°) :: cotangent latitude (54°½) : tangent of ſubſtile's diſtance, which in this caſe = 22° 45′.

II. For the poles height. As radius : coſine of declination :: coſine of latitude : ſine of the height of the pole, = 28° 1′ = ſtile's height.

III. For the plane's difference of longitude. As ſine latitude : radius :: tangent of declination : tangent of difference of longitude = 41° 45′

Now the hour arches before 12 are found by adding continually 15 to 41° 45′, the difference of longitude; but after 12, by continually ſubtracting 15°, till you come to the ſubſtile; beyond which 15° muſt be continually added to the remainder of the hour in which the ſubſtile falls.

If you calculate for quarters, you muſt add and ſubtract 3° 45′ continually. The hour arches being thus determined, the hour angles are had by the following analogy.

As radius : ſine of ſtile's height :: tangent of hour arch : tangent of hour angle. The hour arches and angles in this caſe are as under:

Hours	Hour Arches		Hour Angles		Hours	Hour Arches		Hour Angles	
IX	86°	45′	83°	6	Subſtile	°	′	°	′
X	71	45	54	55	III	3	15	1	32
XI	56	45	35	37	IV	18	15	8	49
XII	41	45	22	45	V	33	15	17	7
I	26	45	13	19	VI	48	15	27	45
II	11	45	5	35	VII	63	15	42	59
Subſtile					VIII	78	15	66	7

The above requiſites being found, take P near the top for the centre of the dial, and draw the meridian PE. Make the angle EPC = 22° 45′, the ſubſtile's diſtance, on the right hand, the declination being weſt; and draw the ſubſtile line PCD. Make the hour angles from the ſubſtile, according to the diſtances in the table, viz. CP IX = 83° 6′; CP X = 54° 55′: CP XI = 35, 37, &c. And thus the hour lines will be formed. Then make the angle CPN = 28° 1′ for the ſtile.

Scholium. The conſtruction would be equally eaſy if the dial declined as much eaſtward; only the forenoon hours, as here deſcribed, would become the afternoon hours on the oppoſite ſide of the dial, and vice verſa; and the ſubſtile which is now placed 3° 15′ before the afternoon hour line of 3, would in that caſe be placed 3° 15′ before the morning hour line of 9. Alſo in finding the hour arches, inſtead of adding or ſubtracting as in the laſt example, the proceſs muſt here be reverſed, uſing ſubtraction in lieu of addition, and addition in the room of ſubtraction.

Case II. *To draw an erect North declining Dial.*

This is done by making a ſouth declining dial, whoſe declination is the ſame and lies the ſame way; for this turned upſide down will be a north declining dial, provided the hours be numbered the contrary way. Thus for a north eaſt decliner make a ſouth eaſt decliner; for a north weſt decliner make a ſouth weſt decliner. The hours before ſun riſe, and ſun ſet muſt be left out. And in drawing the dial the centre H muſt be taken about the middle of the plane, becauſe ſome hours are required here that are not in the ſouth decliner, but theſe will be had by producing the other hour lines through the centre.

SECT. VII.

DIALS INCLINING TO THE HORIZON.

Under this deſcription are all dials whoſe planes are neither horizontal nor vertical, but make with the horizon oblique angles. If the plane of the dial lean forward it is uſually called an incliner; if backward, a recliner. Alſo of each of theſe there are two kinds, viz. direct incliners or recliners; and declining incliners or recliners.

Art. XIII. *To draw a direct inclining or reclining Dial.*

Case I. *To draw a Dial on a plane Parallel to the Equinoctial, or what is uſually called an Equinoctial Dial.*

The plane of this dial reclines from the zenith towards the north in an angle equal to the latitude of the place, and at the poles it becomes an horizontal dial. Wherefore it is only making an horizontal dial by Article I. for the latitude of 90°. In this dial however the hour arches, being equal to the hour angles, which are each = 15°; the dial will be conſtructed by dividing a circle into 24 equal parts, for hour lines, and placing a perpendicular ſtile of any length in the centre.

The ſtile muſt either be a circular pin, of an exceeding ſmall diameter; or otherwiſe an allowance muſt be made for the diameter, by deſcribing a ſmall concentric circle of like radius, and drawing a tangent thereto parallel to the meridian of the plane, for the 12 o'clock hour line, and then the outer circle being divided into parts of 15° each, commencing from the hour line of 12, lines muſt be drawn from each part, to the outer edge of the concentric circle, ſo as to form tangents thereto inclining on each ſide to the 12 o'clock hour line, and to one another, which will be the hour lines required.

N. B. If the dial is conſtructed on the ſouth ſide of the plane, there muſt be hour lines from 6 to 6; but if it is drawn on the north ſide, there need be only the morning and evening hour lines,

lines, viz. from the time of ſun riſe (on the longeſt day) till 6; and and from 6 till the time of the ſun's ſetting. The ſun never ſhines on this ſide of the plane, but when he is in north declination.

Case II. *To draw a Dial for any Latitude on a Plane, paſſing through the Poles of the World, and through the Eaſt and Weſt Points of the Horizon, or which is the ſame, an horizontal Dial under the Equinoctial, uſually called a Polar Dial.*

By Art. X. draw an horizontal dial for 0° latitude or by Art. XI. an erect ſouth dial for the latitude of 90°; or, as the directions in thoſe articles are not ſo well adapted to the preſent caſe, proceed as follows:

By Calculation and the Scale.—As radius is to the ſtile's height, ſo is the tangent of the hour arch, viz. 15°, 30°, 40°, &c. to the diſtance of the hour from the 12 o'clock line. Then thoſe diſtances ſet off on both ſides (*Fig.* 15.) from XII, and C, on the lines AB, and DE, drawn parallel to each other and perpendicular to XII, will give the hour points, and joining I, 1; II, 2; III, 3; &c. and XI, 11; X, 10; IX, 9; &c. they will be the hour lines required; the former being the evening hours, and the latter the morning hours.

N. B. This dial reclines from the zenith towards the north in an angle, equal to the co-latitude.

Case III. *To draw a Dial upon a direct South reclining Plane; or upon a direct North inclining one; whatever may be the Reclination or Inclination.*

1ſt. If it reclines, and the reclination is leſs than the colatitude, add the reclination to the latitude, and the ſum will be the latitude, where the ſaid dial will become an erect South dial, which may therefore be conſtructed by Article II.

2d. If the reclination be greater than the co-latitude, ſubtract the co-latitude from the reclination, and the remainder will be the latitude, where the dial will be horizontal, which may be made by Article I.

3d. For a North inclining dial, make a dial for the South reclining circle of the plane; then the ſame turned up-ſide down, will be a dial for the North inclining plane; obſerving to number the hours the contrary way from the 12 o'clock hour line.

Case IV. *To make a direct North reclining Dial, or a direct South inclining one, whatever may be the Reclination or Inclination.*

1ſt. If the plane reclines, and the reclination is leſs than the latitude, ſubtract the reclination from the latitude, and the remainder will be the latitude where it will become a direct ſouth dial, to be conſtructed by Art. XI. or add the reclination to the colatitude, and the ſum will be the latitude, where it will be an horizontal dial, to be made by Article X.

2d. If the reclination is greater than the latitude, add the complement thereof to the latitude, and the ſum is the latitude for which an horizontal dial muſt be made by Article X.

3d. In ſouth inclining planes, (by Art. XI.) make a direct ſouth dial, for a latitude which is equal to the difference between the given latitude and proclination. If the proclination be greater than the latitude, the dial will belong to ſouth latitude, and muſt be made by Art. X. but the angle of the ſtile will lie the contrary way upon the meridian. A north reclining dial, becomes a ſouth inclining one when turned upſide down.

Case V. *To make an Inclining or Reclining Eaſt Dial.*

By Calculation and the Scale.—The requiſites to be found are, 1ſt. The height of the pole above the plane. 2d. The ſubſtile's diſtance from 12 o'clock. 3d. The plane's difference of longitude, for which we have the following proportions.—1ſt. As radius : coſine of reclination (or proclination) : : tangent of latitude : tangent of the ſubſtile's diſtance from 12 o'clock, running upwards towards the north, or downwards towards the ſouth.—2d. As radius : ſine of latitude : : ſine of reclination (or proclination) : ſine of ſtile's height.—3d. As radius : coſine of latitude : : tangent of reclination, &c. : cotangent of the plane's difference of longitude. To find the hour angles, the analogy is as radius : ſine of ſtile's height : : tangent of the hour arch from the ſubſtile : tangent of the hour angle. The hour arches are found by continually adding and ſubtracting 15° to and from the plane's difference of longitude; the ſaid difference of longitude being one of the hour arches.

Let the lat. be 54°½, and the reclination 21°, then the ſubſtile's diſtance will be 52° 37′; the ſtile's height 16° 58′; and the plane's difference of longitude 77° 26′. Alſo the hour arches and hour angles are as below.

Hours	Hour Arches	Hour Angles	Hours	Hour Arches	Hour Angles
3	57° 34′	24° 40′	8	17° 26′	5° 14′
4	42 34	15 0	9	32 26	10 30
5	27 34	8 40	10	47 26	17 37
6	12 34	3 43	11	62 26	29 12
	Subſtile		12	77 26	52 37
7	2 26	0 43			

The requiſites being found, draw PI (*Fig.* 16.) parallel to the horizon at the bottom of the plane, for the 12 o'clock line; or at the top of the plane if it inclines, in which take any point, as P, for the centre of the dial, toward the left hand. Make the angle IPV on the right hand = ſubſtile's diſtance from the meridian (= 52° 37′) and draw the ſubſtile PV. Make the angle VPT = ſtile's height = 16° 58′, and draw the ſtile PT. Make the hour angles on each ſide of the ſubſtile according to their proper quantities, as found above, and the ſeveral lines will be the hour lines required.

Case VI. *To make an inclining or reclining Weſt Dial.*

This is made as is the eaſt incliner, &c. only the angle PGH, which is equal to the co-latitude muſt be taken on the right hand for a weſt recliner, but to the left hand for an eaſt recliner; and the meridian PI will be at the bottom of the plane, if it reclines; but at the top if it inclines. The hours alſo muſt be reckoned the contrary way in a weſt recliner. N. B. The back ſide of an eaſt incliner, &c. makes a weſt recliner, &c.

Art. XIV. *To draw a Declining, Inclining, or Reclining Dial.*

Case I. *Let it be required to draw a South declining reclining Dial, viz. one whoſe plane declines weſtward 25°, and reclines 15°, in latitude 54½.*

By Calculation and the Scale.—The requiſites to be found, are 1ſt, the height of the meridian; 2d, the height of the pole above the plane; 3d, the diſtance of the ſubſtile from the hour line of 12; and 4th, the plane's difference of longitude.

1ſt. As ſine of reclination : radius : : cotangent declination : tangent of height of the meridian. 2d. As radius : coſine declination : : cotangent of the reclination : tangent of an arch, A; take the difference between A, and the latitude, which call B. If A is leſs than the latitude, your pole is elevated; if greater, the oppoſite pole is elevated.—Then as the cotangent of B : coſine of reclination : : ſine of declination : tangent of the ſubſtile's diſtance from 12. N. B. The ſubſtile runs upwards towards the north in reclining South plane; and their oppoſite incliners. 3d, As coſine of ſubſtile's diſtance : coſine of B : : radius : ſine of ſtile's height; and 4th, As ſine of B : radius : : ſine of ſubſtile's diſtance : ſine of plane's difference of longitude. The plane's difference of longitude will be the hour arch of 12, and the reſt of the hour arches will be had by adding thereto and ſubtracting therefrom 15°, 30°, 45°, &c. and then the hour angles will be found thus. As radius : ſine of ſtile's height : : tangent of hour arch : tangent of hour angle.

In this example the height of the meridian is 83°, 9′; the ſubſtile's diſtance 8°, 1′; the ſtile's height 17°, 19′, and the plane's difference of longitude, 25°, 19′. The hour arches and angles alſo are as under.

Hours	Hour Arches	Hour Angles	Hours	Hour Arches	Hour Angles
7	79° 41′	58° 33′		Subſtile	° ′
6	64 41	32 11	1	10 19	3 7
5	49 41	19 20	12	25 19	8 1
4	34 41	11 38	11	40 19	14 11
3	19 41	6 5	10	55 19	23 16
2	4 41	1 29	9	70 19	39 46
	Subſtile		8	85 19	74 37

Theſe being known, the conſtruction is as follows, draw the horizontal line *b* H (*Fig.* 17.) near the bottom of the plane, in which take a point H, to the right, if the declination be weſt, but to the left if the declination be eaſt. Draw the line ZH, making the angle *b*HZ = 83, the height of the meridian for the 12 o'clock line. Take any point P near the top, when A is greater than the latitude, otherwiſe near the bottom, and draw the ſubſtile PV (to the right for weſt declination) making the angle HPV = 8° 1′, the ſubſtile's diſtance. From P draw the hour lines, making angles with the ſubſtile at P, according to the reſpective hour angle as above found, for the ſeveral hour lines required. Alſo draw PT, making the angle VPT = 17° 9′, the ſtile's height, ſo will PTC be the ſtile or gnomon.

Case II. *To draw a North declining, reclining, or inclining Dial.*

North declining reclining dials may be made, as deſcribed in the laſt caſe; for a ſouth-weſt recliner turned upſide down becomes a north-weſt *incliner*; and a ſouth-eaſt *recliner* turned upſide down, becomes a north-eaſt *incliner*, and vice verſa; the quantities of the declination and inclination remaining the ſame; but the hours muſt be numbered the contrary way.

Alſo if a dial be turned upſide, the backſide thereof (ſuppoſing the hour lines, &c. to appear through the plane) will be its oppoſite incliner.

Having treated on all the various kinds of dials that come under a methodical deſcription of dial planes, I ſhall add ſome uſeful Articles, with another caſe or two of Dialling, not properly coming under the foregoing heads.

SECT. VIII.

USEFUL ARTICLES, &c.

ART. XV. *Having the Situation of any direct South inclining Dial Plane, to find the Hour of the Day when the Sun leaves the North Side to shine on the South Side thereof.*

Say, as radius : tangent of sun's declination : : tangent of the height of the pole above the plane : cosine of the hour angle at the pole from noon which reduced into time (by allowing 15° to an hour) shews how long before or after noon, the sun begins to shine on the South side.

ART. XVI. *To find the Hour of the Day when the Sun leaves one Side of a Vertical declining Plane to shine on the other.*

Find the time from noon (by the last Article) when the sun leaves one side of a direct north or south plane to shine on the other; then, if the plane declines east, to the time so found, add the plane's difference of longitude in time, and the sum will be the distance of time before noon, when the sun leaves the north side. Also subtract the plane's difference of longitude from the time found, and you will have the time of leaving the South side.—But if the plane declines west, the said difference will shew the distance of time before noon, when the sun begins to shine on the south side; and the sum will be the time when he leaves the south side.

ART. XVII. *To find the Hour of the Day when the Sun goes off the East Side of an East reclining Dial to shine on the West Side.*

Say, as radius : tangent of the sun's declination : : tangent of the poles height above the plane, to the cosine of the hour angle from 12, at the pole, which will be greater than 90°, when the sun's declination is North. Then subtract the plane's difference of longitude from this angle, and the remaining angle, converted into time, gives the hour after 12 at noon, when the sun leaves the East side.

ART. XVIII. *To find the Hour on a given Day, when the Sun comes on or goes off the South Side of a South reclining Plane, declining also East or West.*

Find the height of the pole above the plane, and the plane's difference of longitude; and also the hour angle, as if it was a direct South plane, by Article XV. to which add the plane's difference of longitude, and it will give the hour angle from noon when the sun comes upon the given plane, if an East decliner; also from the hour angle subtract the plane's difference of longitude for the time of his leaving the plane; the reverse of these cases will give the hour angles from noon for an East decliner.

ART. XIX. *Having a North reclining Dial Plane, declining East or West, to find the Hour of any Day when the Sun goes upon the North Side.*

The hour angle being found, as in the last Article, the difference between it and the plane's longitude reduced to time, will shew the distance of time from noon, when the sun comes upon the north side of a west decliner, which is before noon, when the plane's longitude is the greater, otherwise after noon. The direct contrary of this obtains in an East decliner.

ART. XX. *To find what Latitude a Dial is made for.*

Measure the angle between 12 and any hour line; take the hour arch belonging to it, and say, As the tangent of the hour arch : tangent of the hour angle : : radius : sine of latitude in an horizontal dial, or the cosine of latitude for a direct south or north dial.

ART. XXI. *To find the Hour of the Night by the Moon's shining upon a Sun-Dial.*

From an ephemeris, or almanack, take the time of the moon's southing. Observe how many hours and minutes the shadow of the dial wants of 12 o'clock, and subtract them from the time of southing for the hour of the night. But if the shadow be after 12, add the said hours and minutes to the time of her southing; rejecting 12, if it exceed, and you will have the hour of the night.

SCHOLIUM. In fixing any dial, the three following directions must be observed. 1st. That the horizontal line be placed parallel to the horizon. 2d. That it be placed so as to have its proper declination; and 3d. That it may also have its proper degree of reclination or proclination.

ART. XXII. *To draw all the Primary Dials, on the same Block or Post.*

1. Let the plane ABCD (*Fig.* 18.) in the proper position of the block, be supposed horizontal; and thereon describe a horizontal dial.

2. Draw the right lines EM and FL parallel to DC, which, accordingly, in the proper position of the block, will be parallel to the horizon: then let the plane BNMC make an angle with EM, equal to the elevation of the pole CME; and thereon describe an upper polar dial.

3. Let the opposite plane, ADE, make with EM an angle DEM, equal to the elevation of the equator; and on this draw an upper equinoctial dial.

4. Let the plane, KLHI, make with FL an angle HLF, equal to the elevation of the equator; and on this inscribe a lower equinoctial dial.

5. Let the opposite plane, FG, make with FL an angle, GFL, equal to the elevation of the pole; and here draw a lower polar dial.

6. Let the plane MNKL, and the opposite one EF, be perpendicular to FL; and on that draw a south dial, and on this a north dial.

7. On the plane EMLF describe a west dial; and on the opposite plane an east dial.

If, then, the block be so placed, as that the plane, MNLK, looks to the south, and the plane of the meridian bisect it in the line of 12 o'clock in the horizontal dial ABCD, and south dial MNKL, all the hours of the day will be indicated by several planes at once.

ART. XXIII. *To describe an Universal Equinoctial Dial,* (*Fig.* 19.)

Join two metal, or ivory planes, ABCD, and CDEF, so as to be moveable at the joint. On the upper surface of the plane ABCD, describe an *upper equinoctial dial,* and upon the lower a *lower,* as already directed; and through the centre I drive a stile. In the plane DEFC cut a box, and put a magnetical needle G therein: fit on the same plane a brass quadrant nicely graduated, and passing through a hole cut in the plane ABCD. Now, since this may be so placed, by means of the needle, as that the line I 12 shall be in the plane of the meridian; and, by means of the quadrant, may be so raised, as that the angle BCF shall be equal to the elevation of the equator; it will serve as a dial in any part of the world. When the sun is in the equator, these dials can be of no use.

ART. XXIV. *To make a Dial on the Surface of a Sphere.* (*Fig.* 20.)

Let AEBQ be a sphere. Mark two opposite points therein for the poles, of which P is one. At an equal distance from the poles describe the equinoctial EQ, which divide into 24 equal parts for hours. Then let the point which is on the meridian PB be marked with 6; and the other hour points as in the figure. Those placed above the equinoctial are the forenoon hours; and those below the equinoctial the afternoon hours. The globe must be fixed so in the sun that the pole P may be elevated above the horizon, so that its axis may point directly to the pole, and the point 6 must be on the top, so that the circle P 6 B may be in the meridian. Then as the sun constantly illuminates half the globe, the circle, terminating the enlightened part, will shew the hour of the day. And 12 o'clock will be shewn at two places, namely, at E and Q.

ART. XXV. *To draw a Dial upon the Ceiling of a Room, that will shew the Hour by Reflection.*

Place a small piece of a looking glass exactly horizontal in a window, on which the sun shines, measure the perpendicular distance of the glass from the ceiling, for the height of the stile, and where the perpendicular cuts the sealing is the foot of the stile. Then having the height of the stile and its foot, make an horizontal dial thereto upon the ceiling by ART. X. and it is done.

N. B. Instead of a glass you may use a little water, which, if a glass is not perfectly horizontal, will be more correct; for water of itself will have its surface horizontal; also water being always in motion by the agitation of the air, will cause the point of reflection on the ceiling to be the more easily distinguished. When a glass is used, the error, if any, in its horizontal position, will be doubled by reflection.

ART. XXVI. *To draw Meridian Lines in a Dial, to shew when it is Noon, at any particular Places on the Earth.* (*Fig.* 21.)

The meridian of any place is easily drawn in a dial, if the longitude thereof be known. Thus reduce the difference of longitude between that place and the place for which the dial is made into time, and reckon so many hours and minutes from 12 upon your dial towards the west, if the place lies east; or towards the east if the place is west: then from the point to which they reach, draw a line to the center; and when the shadow falls upon such line, the sun is in the meridian of the said place; or having the difference of longitude, or the hour arch, the hour angle may be found, as has been before shewn.

In digesting this Treatise, I have selected, from the most approved authors, every thing that appeared valuable; and notwithstanding I have studied brevity, have fully enlarged where the subject required. And as a necessary introduction, have treated amply on the Gnomonic Projection. In this Treatise I have consulted various Authors on the subject, particularly Emerson; nor have I omitted any thing that could illustrate the art. I have also been studious to select the best constructions and modes of calculation, and given copious descriptions of those Dials which are of most general use; so that I have brought into this compass a complete Practical System.

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